**Title: Applying the Monte Carlo Method to a Counting Problem in the Game, *Battleship***

**Brief Overview:**

Combinatorics is a branch of mathematics which is chiefly concerned with counting. It is rich with problems that are easy to state, yet difficult to solve. One such problem is determining the number of setup positions in the game, *Battleship*. Using the principle of inclusion/exclusion, we transform the problem into computing a probability. Using a “Monte Carlo” simulation, we estimate this probability (and hence the number of setups) to within one percent.

In this simulation activity, students are given the opportunity to investigate and apply mathematical concepts through the construction of their own solution algorithm. The activity and related mathematics are not computer-language dependent. The emphasis has been placed on problem solving and the related mathematics, rather than on a specific programming language.

**Links to NCTM 2000 Standards:**

- **Mathematics as Problem Solving**
  Students will discover that one of the most significant applications of computers is simulation. Methods implemented in programming algorithms allow individuals to investigate patterns of a process without actually conducting the experimental procedure. Furthermore, programming algorithms can serve as a key tool in investigating applications which are otherwise inaccessible to many.

- **Mathematics as Reasoning and Proof**
  Students must determine how the mathematical properties and procedures related to the problem under consideration may be implemented and justified in a simulation algorithm.

- **Mathematics as Communication**
  Students must propose and justify their solution strategies for estimating the related theoretical probability to their peers in a verbal brainstorming session. In addition, they may be required to submit a written presentation of their procedure and/or class procedure.

- **Mathematics as Connections**
  Students will use concepts of mathematics in a modeling/simulation process.

- **Mathematics as Representation**
  Students will use a computer generated model to represent, simulate, and interpret the findings for the problem under consideration.

- **Data Analysis, Statistics, and Probability**
  Students will apply counting procedures and relative frequency techniques in the development of a simulation algorithm to approximate the value of a theoretical probability.

**Links to Maryland High School Mathematics Core Learning Goals:**

**Data Analysis and Probability**

- **3.1.1**
  Students will design and/or conduct an investigation that uses statistical methods to analyze data and communicate results.

- **3.1.3**
  Students will use simulations to make statistical inferences from data to estimate the probability of an event.

**Grade/Level:**

11-12

**Duration/Length:**

One hour to present and discuss the activity. Completion time of the related program simulation will vary with the background of the students.

**Prerequisite Knowledge:**

Students should have working knowledge of the following skills:

- Familiarity with a random number generator in the computer language of the student's choice
- A thorough understanding of ratios

**Student Outcomes:**

Students will be able to:

- apply combinatoric concepts in similar simulation algorithms.
- apply "Monte Carlo" techniques in similar simulation algorithms.

**Materials/Resources/Printed Materials:**

- A computer or programmable calculator.

**Development/Procedures:**

- In the game, *Battleship*, each player begins by placing five ships on a 10x10 grid. The ships must be oriented either horizontally or vertically and have respective lengths of 5, 4, 3, 3, and 2. The rules state that no two ships can share a common square, i.e., no overlaps are allowed. We would like to enumerate the number of possible setups for a single player, but a complete solution is not so easy to calculate. We first apply the principle of inclusion/exclusion to frame the problem in terms of finding a probability.

- Even though it is not the number we are seeking, consider the number of possible positions which **include** overlaps. The 5-long ship has 6 possible horizontal placements in the first row, and there are 10 rows, for a total of 60 horizontal placements. Similarly, there are 60 vertical placements and hence a total of 120 placements for the 5-long ship. Similarly, there are 140 placements of the 4-long ship, 160 placements for each of the 3-long ships, and 180 placements for the 2-long ship for a grand total of

$$T = 120 \times 140 \times 160 \times 160 \times 180 = 77{,}414{,}400{,}000$$

  possible setups if overlaps are allowed. From these setups we would like to **exclude** those setups which have overlaps. Let us call such setups **illegal**, and let I denote the number of illegal setups. Those setups which do not have overlaps will be called **legal**, and let L denote the number of legal setups.

Then

$$T = L + I$$

and the ratio

$$P = \frac{L}{T}$$

is the probability that a random setup (where overlaps are allowed) is legal.

- Loosely speaking, a Monte Carlo method is any method which can obtain non-random information by studying a suitable random process. While the name Monte Carlo was coined by mathematicians at the Los Alamos Laboratory in the 1940s, such methods have been used as early as 1777 by Buffon in his famous needle-dropping experiment.

- In our situation we use the Monte Carlo method to estimate the probability P by taking a large sample of random setups (with overlaps allowed) and counting the number of those setups which do not have overlaps. The ratio of this count to the number of samples gives us an estimate of P (which is 0.38 to two significant figures), and hence an estimate of L.

**Procedures:**

- Write a program to implement the Monte Carlo simulation described above and obtain an estimate of the number of legal setups in *Battleship*. One strategy is to keep the information in a 10x10 matrix which for each trial is initialized to all zeros. Entries of this matrix are incremented whenever a ship is placed. When ships overlap some entry will be at least 2.

**Assessment:**

The students will submit a correct program, flowchart, and discussion of the solution strategy implemented in the simulation algorithm.

- ***Level 3:*** The presentation offers clear evidence of a thorough understanding of the mathematics related to the activity.

  - Simulation program is correct.
  - Discussion of the strategy for the solution algorithm is correct and consistent with the flowchart submitted.
  - Correct and consistent symbolism is present in the flowchart submitted.

- ***Level 2:*** The presentation offers evidence of substantial understanding of the mathematics related to the activity.

  - Discussion of the strategy for the solution algorithm is correct but is consistent with the flowchart submitted, ***or***
  - The flowchart submitted depicts a correct strategy but no complete, consistent, and correct written discussion is submitted.

- ***Level 1:*** The presentation offers a reasonable understanding of the necessary structure of a correct solution strategy.

## Extension/Follow Up:

- Do several trials of varying sizes. Why can't we get any closer than one percent?
- There are many parameters to vary: board size, number of ships, lengths of ships, allowable orientations, dimensions of board and ships, etc.
- By making a judicious use of symmetries, calculate the actual number of possible setups by brute force. (There are 30,093,975,536).
- Invite a professional mathematician or computer scientist to make a related presentation.

## Authors:


Carvel LaCurts
Pocomoke High School
Worcester County, MD

David Fu
National Security Agency
Fort Meade, MD


## References:


*Battleship, The Classic Naval Combat Game*. Milton Bradley Company of Hasbro, Inc. 1990, 1996.